Rising AP Calculus AB Summer Assignment

All students how have completed Precalculus and are enrolled in **AP Calculus AB** should complete the following assignment over the summer. Please note the directions at the beginning of each section in addition to the example problems that accompany each. This assignment should take, at most, five hours to complete. If there are particular parts that give you difficulty, please consult your notes and examples from your Precalculus materials and/or consider an online resource (such as Khan Academy).

This assignment is NOT intended to stress you our and cause undo heartache on you over the summer. It is simply a reminder of skills that you will be expected to recall with ease as we progress through the Calculus course. This assignment is due on the first day of class when we return in August.

Should you have any questions, you may contact the math department chair, Mr. Divers, at ddivers@rvgs.k12.va.us or the Calculus AB instructor, Mr. Taylor at bmtaylor@rvgs.k12.va.us.

Good luck and have a wonderful summer!

Student's Name_____

Summer Packet for Students entering AP Calculus
(Students who have successfully completed Math Analysis/Trig or a similar PreCalculus course)
Show your work throughout this Summer Packet!

Use the rules of logarithms to simplify expressions and solve equations.
Give exact answers such as in the second example. No calculator rounded decimals.

<table>
<tr>
<td>Example:
$2 \log x + 3 \log y - \log x$
$\log x^2 + \log y^3 - \log x$
$\log x^2 y^3 - \log x$
$\log \frac{x^2 y^3}{x}$
$\log x y^3$</td>
<td>Example:
$e^{3x+6} = 27$
$\ln e^{3x+6} = \ln 27$
$3x + 6 = \ln 27$
$3x = \ln 27 - 6$
$x = \frac{\ln 27 - 6}{3}$
$x = \frac{1}{3}\ln 27 - 2$
$x = \ln 27^{\frac{1}{3}} - 2$
$x = \ln 3 - 2$</td>
<td>1.
$3 \ln x + \ln y$</td>
</tr>
<tr>
<td rowspan="2">3.
$2 \log 6 + 3 \log 2 - \log 3$</td>
<td rowspan="2">4.
$0.5 \ln 9$</td>
<td>2.
$2 \ln x - \frac{1}{2}\ln x$</td>
</tr>
<tr>
<td>6.
$\log 1$</td>
</tr>
<tr>
<td></td>
<td>5.
$\ln 1$</td>
<td>7.
$\ln e^3 + \log 100$</td>
</tr>
<tr>
<td>8.
$e^{2x+4} = 25$</td>
<td>9.
$\ln x - 3\ln x + 4\ln y - \frac{1}{2}\ln 9$</td>
<td>10.
$e^{3x+5} = e^{5x+2}$</td>
</tr>
</table>

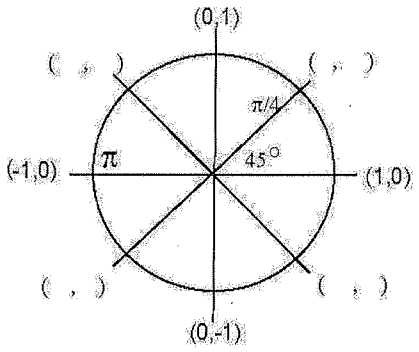
Student's Name_____

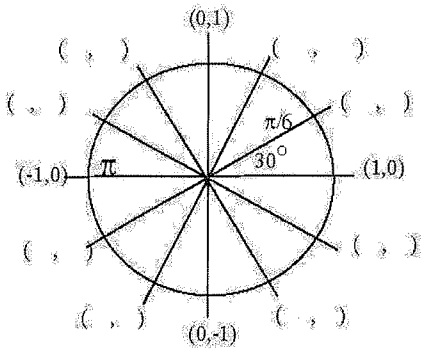
Find an equation for each line. Use the formula $y - y_1 = m(x - x_1)$ for each problem.

Example: Slope = 3, through (2,5). $y - -5 = 3(x-2)$ $y + 5 = 3x - 6$ $y = 3x - 11$	Example: Perpendicular to $y = \frac{-2}{3}x - 8$, through (4,7). $y - 7 = \frac{3}{2}(x-4)$ $y - 7 = \frac{3}{2}x - \frac{12}{2}$ $y - 7 = \frac{3}{2}x - 6$ $y = \frac{3}{2}x + 1$	Example: Parallel to $4x - 3y = 7$, through (2,0). $4x - 3y = 7$ $-3y = -4x + 7$ $y = \frac{4}{3}x + \frac{-7}{3}$ $y - 0 = \frac{4}{3}(x-2)$ $y = \frac{4}{3}x - \frac{8}{3}$
11. Slope = 5, through (-3, -2).	12. Perpendicular to $y = \frac{3}{5}x + 6$, through (6, 2).	13. Parallel to 3x + y = 8, Through (-2, -1).
14. Through (5, 2) and (-3, 6).	15. Slope = $\frac{2}{7}$, through (6, 0).	16. Through (-3, -5), and (0, -8).

Student's Name_____

Questions 17-21, Fill in the coordinates below in simplified radical form.





Complete the coordinates for the terminal positions of the indicated angles drawn on these unit circles. Write the radian measure on the terminal side of each angle in standard position, like $\pi/6, \pi/4, \pi$.

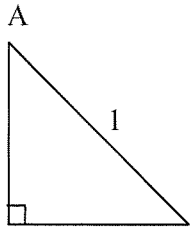
Complete the right triangle definitions of the basic trigonometric ratios.

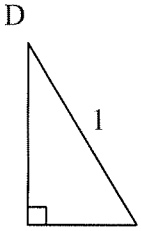
22. $\sin\theta =$ _____

$\cos\theta =$ _____

$\tan\theta =$ _____

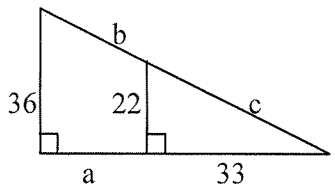
Supply the missing sides of the 45-45-90 and 30-60-90 right triangles.





23. AC = ____ CB = ____ 24. DE = ____ EF = ____

25. Find a, b, and c.



26. Solve:

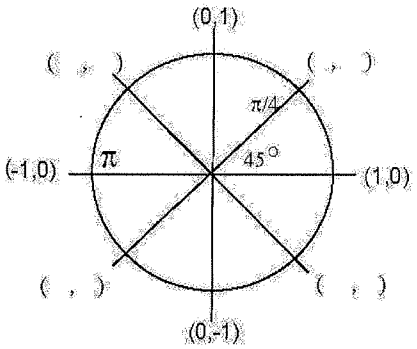
$$2x^2 - 4x = 6$$

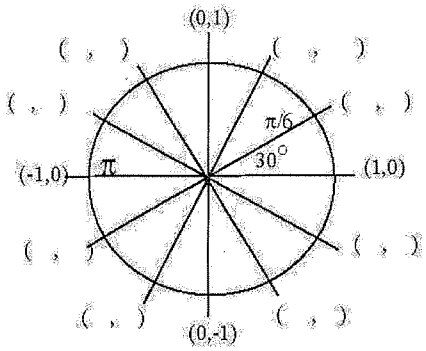
27. Solve:

$$3x^2 - 8x + 2 = 0$$

Student's Name_____

The unit circles below are for reference only. You may fill in angles and coordinates if you wish.





Complete the coordinates for the terminal positions of the indicated angles drawn on these unit circles. Write the radian measure on the terminal side of each angle in standard position, like $\pi/6, \pi/4, \pi$.

Use the following unit circle coordinate definitions of the six trig functions to express the following trig ratios in simplified radical form.

$\sin\theta = y \quad \csc\theta = \frac{1}{y}$

$\cos\theta = x \quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$

Example: $\sin\frac{3\pi}{2} = ?$

$\sin\theta = y$, $\sin\frac{3\pi}{2} = -1$

Example: $\sec\frac{\pi}{6} = ?$

$\sec\theta = \frac{1}{x}$

$\sec\frac{\pi}{6} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Example: $\tan\frac{\pi}{6} = ?$

$\tan\theta = \frac{y}{x}$

$\tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

28. $\sin\frac{5\pi}{4} = ?$

29. $\sec\frac{2\pi}{3} = ?$

30. $\tan\frac{\pi}{3} = ?$

31. $\csc 2\pi = ?$

32. $\cos\frac{11\pi}{6} = ?$

33. $\cot\frac{7\pi}{4} = ?$

34. $\csc\frac{3\pi}{4} = ?$

35. $\cot\frac{\pi}{2} = ?$

36. $\sin\frac{7\pi}{6} = ?$

Student's Name_____

Draw a diagram and set up an appropriate trig ratio to solve the following problems. Make sure that your calculator is set to the correct MODE, radian or degree.

37. An airplane is flying at an altitude of 4000 feet above the ground. The pilot sights the beginning of the runway at an angle of depression of $\frac{\pi}{6}$ radians. What is the slant range from the airplane to the beginning of the runway.

38. A man standing on top of a 100 ft building sites the top of a nearby building at an angle of elevation of $\frac{\pi}{8}$. If he knows the nearby building is 160 feet high, what is his horizontal distance from that building.

39. A tree casts a 48 ft shadow when the sun's elevation in the sky is $\frac{3\pi}{5}$ radians. How tall is the tree?

40. An isosceles triangle has base angles of 65° and a base of 14 km. Calculate the height of the triangle, and then determine the area of the triangle. Identify the units in your answers.

41. A right triangle has legs of 14 cm and 23 cm. Determine the degree measure of the angle opposite the 14 cm leg. (Round answer to the nearest tenth).

42. Write the following sentence as an "if...then..." conditional. Then write its' inverse, converse, and contrapositive sentences. Identify each of the 4 sentences as true or false.

All rectangles are parallelograms.

Student's Name_____

Use the following functions to answer the questions on this page and the next page. Write the answers for domain and range in interval notation.

$f(x)=\sqrt{3x-7}-4$ $\quad k(x)=\frac{4x+8}{x^2-5x-24}$ $\quad p(x)=\ln(6x-3)$

$g(x)=|2x-6|-5$ $\quad m(x)=\sqrt{5x+8}-3$ $\quad r(x)=\frac{3x+2}{x^2-4}$

$h(x)=\ln(-5x+2)$ $\quad n(x)=|2x-4|-1$ $\quad s(x)=\frac{x^2-9}{x^2+7x+10}$

Example: What is the domain of f? $3x-7\geq 0$ $3x\geq 7$ $x\geq\frac{7}{3}$ $\left[\frac{7}{3},\infty\right)$ What is the range of f? $\sqrt{0}-4=-4$ $[-4,\infty)$	Example: What are the zeros of f? $\sqrt{3x-7}-4=0$ $\sqrt{3x-7}=4$ $\left(\sqrt{3x-7}\right)^2=4^2$ $3x-7=16$ $3x=23$ $x=\frac{23}{3}$	Example: What are the zeros of g? $\lvert 2x-6\rvert-5=0$ $\lvert 2x-6\rvert=5$ $2x-6=5 \quad 2x-6=-5$ $2x=11 \quad 2x=1$ $x=\frac{11}{2} \quad x=\frac{1}{2}$
Example: What is the domain of h? $-5x+2>0$ $-5x>-2$ $x<\frac{2}{5}$ $\left(-\infty,\frac{2}{5}\right)$ What is the range of h? $(-\infty,\infty)$ What are the zeros of h? There are none.	Example: What is the domain of k? $x^2-5x-24\neq 0$ $(x-8)(x+3)\neq 0$ $x-8\neq 0 \quad x+3\neq 0$ $x\neq 8 \quad x\neq -3$ $(-\infty,-3)\cup(-3,8)\cup(8,\infty)$ What are the zeros of k? $4x+8=0$ $4x=-8$ $x=-2$	43. What is the domain of m? 44. What is the range of m?
45. What are the zeros of m?	46. What is the domain of n? 47. What is the range of n?	48. What are the zeros of n?

Student's Name_____

Use the following functions to answer the questions on this page. Write the answers for domain and range in interval notation.

$f(x)=\sqrt{3x-7}-4$ $\quad k(x)=\frac{4x+8}{x^2-5x-24}$ $\quad p(x)=\ln(6x-3)$

$g(x)=|2x-6|-5$ $\quad m(x)=\sqrt{5x+8}-3$ $\quad r(x)=\frac{3x+2}{x^2-4}$

$h(x)=\ln(-5x+2)$ $\quad n(x)=|2x-4|-1$ $\quad s(x)=\frac{x^2-9}{x^2+7x+10}$

49. What is the domain of p ?	50. What are the zeros of p ?	51. What is the domain of r ?
52. What are the zeros of r ?	53. What is the domain of s ?	54. What are the zeros of s ?
Example: Find f^{-1}, the inverse of f. $y=\sqrt{3x-7}-4$ *exchange _ x _ and _ y* $x=\sqrt{3y-7}-4$ $x+4=\sqrt{3y-7}$ $x^2+8x+16=3y-7$ $x^2+8x+23=3y$ $y=\frac{x^2+8x+23}{3}$ $f^{-1}(x)=\frac{x^2+8x+23}{3}$	55. Find m^{-1}.	56. Find p^{-1}.

Student's Name_____

Use the following functions to answer the questions on this page.

$f(x) = 3x - 6$ $\quad j(x) = x - x^3$

$g(x) = e^{x+4}$ $\quad k(x) = 3x$

$h(x) = x^2 - 5$ $\quad m(x) = \frac{x^2}{x-3}$

Example: Find $f(4)$. $f(4) = 3 \cdot 4 - 6$ $= 12 - 6$ $= 6$ Example: Find $(f \cdot h)(2)$. $(f \cdot h)(2) = f(2) \cdot h(2)$ $= (3 \cdot 2 - 6)(2^2 + 5)$ $= (6 - 6)(4 + 5)$ $= (0)(9)$ $= 0$	Example: Find $(k - j)(z^2)$. $(k - j)(z^2) = 3(z^2) - [(z^2) - (z^2)^3]$ $= 3z^2 - [z^2 - z^6]$ $= 3z^2 - z^2 + z^6$ $= 2z^2 + z^6$	Example: Find $[m \circ k](x)$. $[m \circ k](x) = m(k(x))$ $= m(3x)$ $= \frac{(3x)^2}{(3x)-3}$ $= \frac{9x^2}{3x-3}$ $= \frac{9x^2}{3(x-1)}$ $= \frac{3x^2}{x-1}$
57. Find f^{-1}.	58. Find $m(-4)$.	59. Find $(f \cdot j)(x)$.
60. Find $(h \circ k)(x)$.	61. Find $(g \circ f)(x)$.	62. Find $(j \div k)(x)$.
63. Find $m(b+3)$.	64. Find $(m \circ k)(x)$.	65. Find $(h \circ g)(x)$.

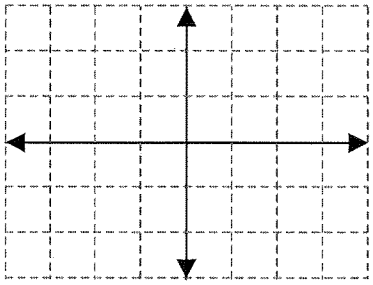
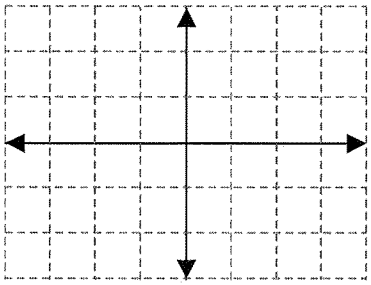
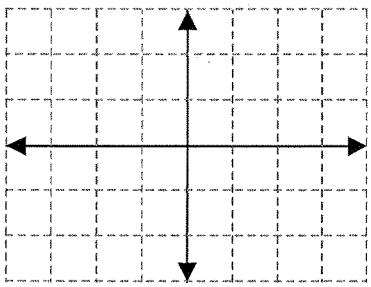
Student's Name_____

Use the following functions to answer the questions on this page.

$f(x) = 3x - 6$ $\quad j(x) = x - x^3$

$g(x) = e^{x+4}$ $\quad k(x) = 3x$

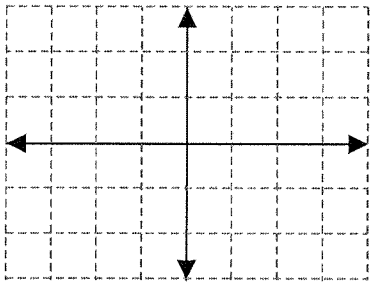
$h(x) = x^2 - 5$ $\quad m(x) = \frac{x^2}{x-3}$

66. Find $(g \div f)(-4)$.	67. Find $(f + k)(5)$.	68. Find $(h \circ m)(2)$.
69. Find $(k \circ j)(-2)$.	70. Find $m(3)$.	71. What are the zeros of j ?
72. Where do f and h intersect?	73. Where do f and k intersect?	74. Where do j and k intersect?
75. Graph $y = x$ 	76. Graph $y = x^2$ 	77. Graph $y = x^3$ 

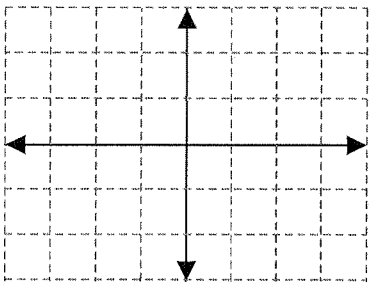
Student's Name_____

The three graphs on the previous page and all the graphs on this page should be graphed from memory, without the use of a graphing calculator. The last three require some knowledge of basic transformations, specifically shifting left or right and shifting up or down. **Label axes.**

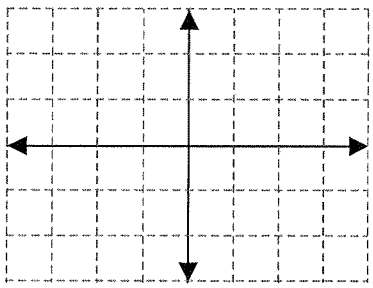
78. Graph $y=\sqrt{x}$



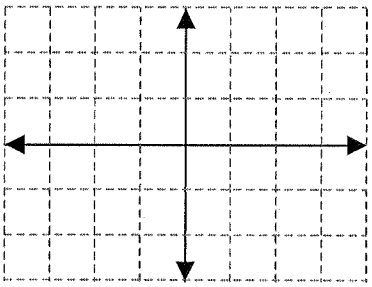
79. Graph $y=\frac{1}{x}$



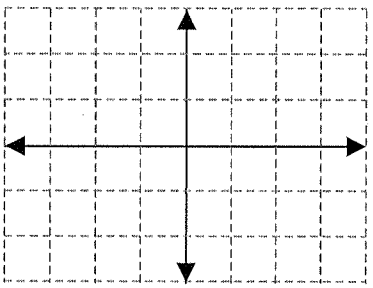
80. Graph $y=|x|$



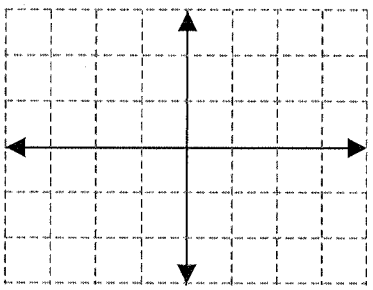
81. Graph $y=e^{x}$



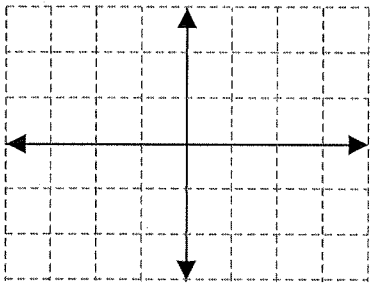
82. Graph $y=\ln x$



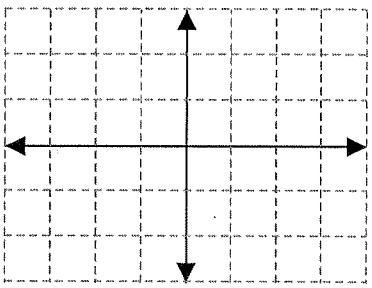
83. Graph $y=\sin x$, $[0,2\pi]$



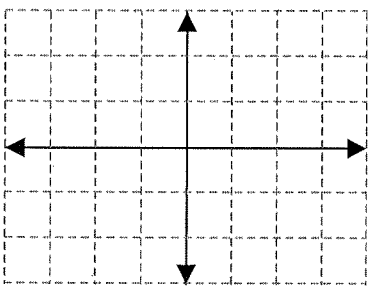
84. Graph $y=\cos x$, $[0,2\pi]$



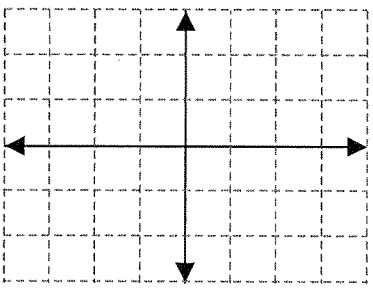
85. Graph $y=\tan x$, $[\frac{-\pi}{2},\frac{\pi}{2}]$



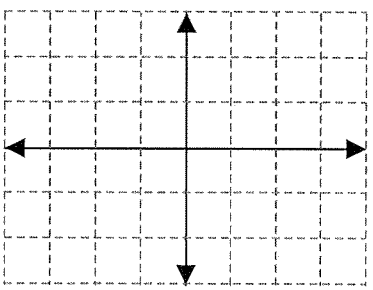
86. Graph $y=e^{x-2}$



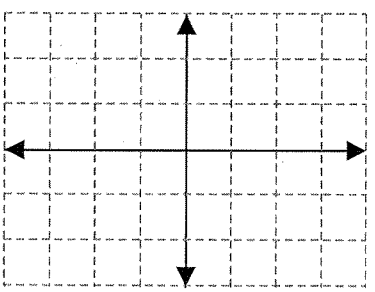
87. Graph $y=|x-2|-1$



88. Graph $y=(x+1.5)^{3}$



89. Graph $y=2+\sqrt{x+1}$



Student's Name_____

Simplify each rational expression. Make sure there is only one fraction in your answer (other than exponents) and there are no negative exponents in your answer.

Example: $\frac{3x^2-6x-24}{3x^2+2x-8}$ $\frac{3(x^2-2x-8)}{(3x-4)(x+2)}$ $\frac{3(x-4)(x+2)}{(3x-4)(x+2)}$ $\frac{3x-12}{3x-4}$	Example: $\frac{3x^{\frac{1}{2}}+2x^{\frac{3}{2}}-x^5}{x^{\frac{5}{2}}}$ $\frac{x^{\frac{1}{2}}(3+2x-x^{\frac{9}{2}})}{x^{\frac{5}{2}}}$ $\frac{3+2x-x^{\frac{9}{2}}}{x^2}$	Example: $\frac{3x^{-2}+\frac{5}{4}x-\frac{1}{4}x^{-1}}{(3x^2)^2}$ $\frac{\frac{1}{4}(12x^{-2}+5x-x^{-1})}{9x^4}$ $\frac{\frac{1}{4}x^{-2}(12+5x^3-x)}{9x^4}$ $\frac{12+5x^3-x}{36x^6}$
90. $\frac{x^2-25}{x^2-12x+35}$	91. $\frac{5x^{\frac{3}{2}}+10x^{\frac{1}{2}}-5x^{\frac{5}{2}}}{10x^{\frac{5}{2}}}$	92. $\frac{4x^2-12x-40}{x^2-8x+15}$
93. $\frac{3x^{\frac{-1}{2}}+4x^{\frac{-3}{2}}-5x^{\frac{1}{2}}}{6x^{\frac{5}{2}}}$	94. $\frac{4e^{-x}-6e^{x}+2e^{2x}}{(e^{-x})^2}$	95. $\frac{4x^{-2}+2x-6x^{-1}}{8x^2}$
96. $\frac{\frac{1}{3}x^2-5x+\frac{2}{3}x^4}{4x^3}$	97. $\frac{3xy^{-1}+4x^{-1}y-5x^2}{2xy}$	98. Find the midpoint of, the slope of, and the distance between the points (3,-6) and (-5,-2).