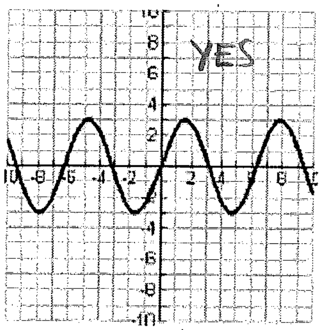
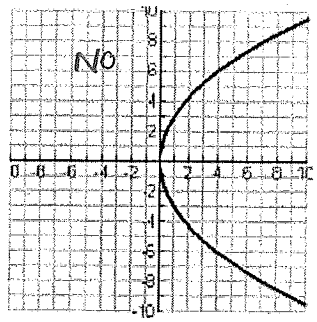
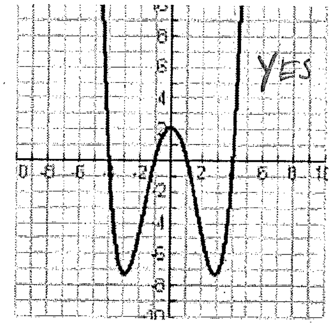
Functions

1. Determine whether each of the following are graphs of functions:

a. 

b. 

c. 

2. Find the domain and range of the functions below. Use interval notation.

a. $f(x) = \frac{1}{x-4}$
D: $(-\infty, 4) \cup (4, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

b. $g(x) = \sqrt{2x-5}$
D: $[2.5, \infty)$
R: $[0, \infty)$

c. $h(x) = 2^x + 3$
D: $(-\infty, \infty)$
R: $(3, \infty)$

d. $f(x) = \log(x)$
D: $(0, \infty)$
R: $(-\infty, \infty)$

e. $g(x) = 9 - x^2$
D: $(-\infty, \infty)$
R: $(-\infty, 9]$

f. $h(x) = |x-2|$
D: $(-\infty, \infty)$
R: $[0, \infty)$

3. If $f(x) = 3x - 1$ and $g(x) = x^2 + 2x$, evaluate the following:

a. $f(g(2)) = f(8) = 23$

b. $f(f(5)) = f(14) = 41$

c. $f^{-1}(17) = 6$
$* f^{-1}(x) = \frac{x+1}{3}$

d. $g(f(x)) = (3x-1)^2 + 2(3x-1)$
$= 9x^2 - 6x + 1 + 6x - 2$
$= 9x^2 - 1$

e. $f(a+1)$
$= 3(a+1) - 1$
$= 3a + 2$

f. $g(x+h) = (x+h)^2 + 2(x+h)$
$= x^2 + 2xh + h^2 + 2x + 2h$

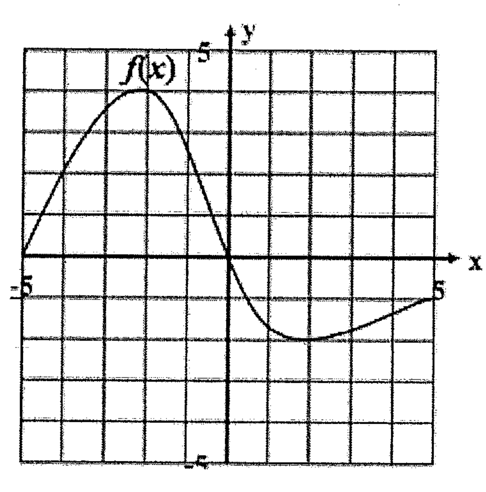
4. Use the graph at the right to answer the questions.

a. Find $f(-2)$ $= 4$

b. Find $f(5)$ $= -1$

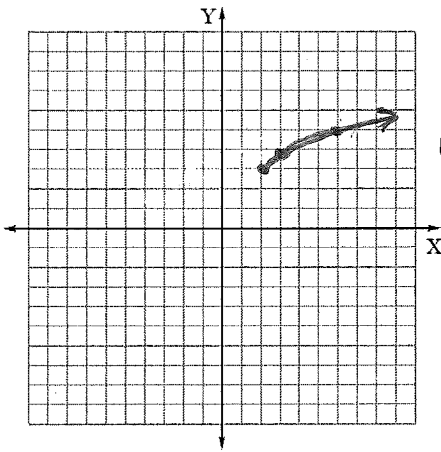
c. For which value(s) of x does $f(x) = -2$?
$x = 2$

d. For which value(s) of x does $f(x) = 0$?
$x = -5, x = 0$



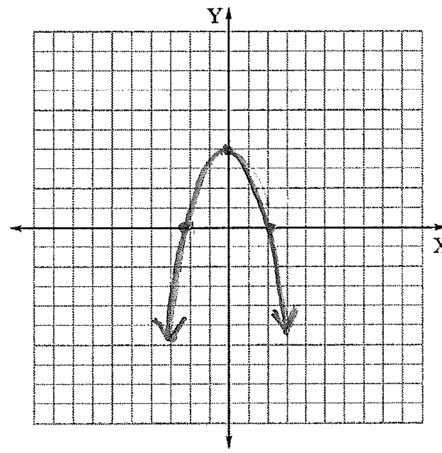
5. Sketch an accurate graph. Show at least 3 points. Name all intercepts and asymptotes.

a. $f(x) = \sqrt{x-2} + 3$



NO INTs

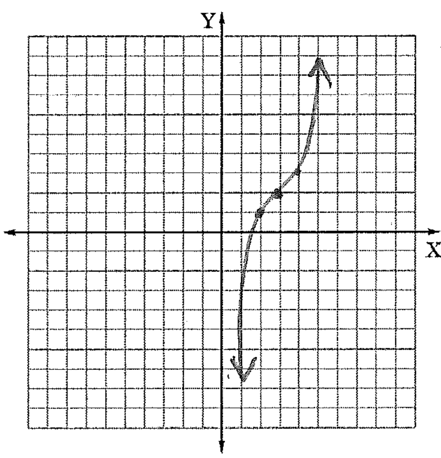
c. $f(x) = 4 - x^2$



y-INT: $(0,4)$

x-INT: $(-2,0)$
$(2,0)$

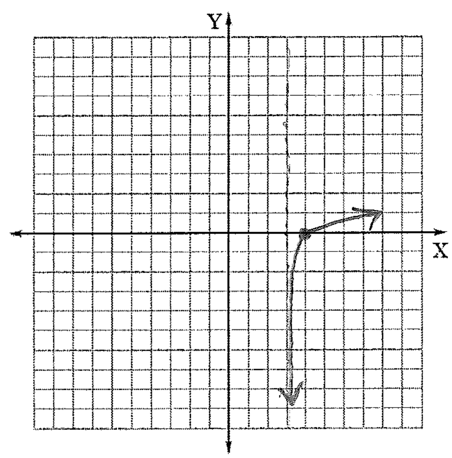
b. $f(x) = (x-3)^3 + 2$



y-INT: $(0,-25)$

X-INT: $(3-\sqrt[3]{2}, 0)$

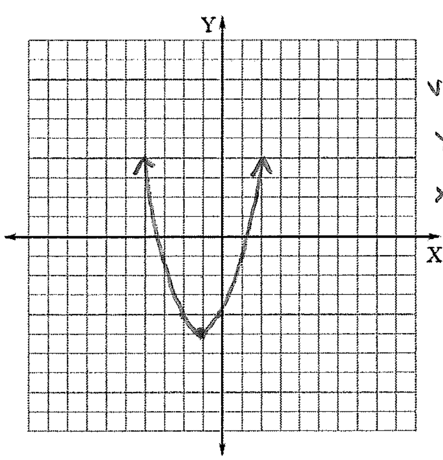
d. $f(x) = \log(x-3)$



VA @ $x=3$

x-int: $(4,0)$

e. $f(x) = (x+1)^2 - 5$

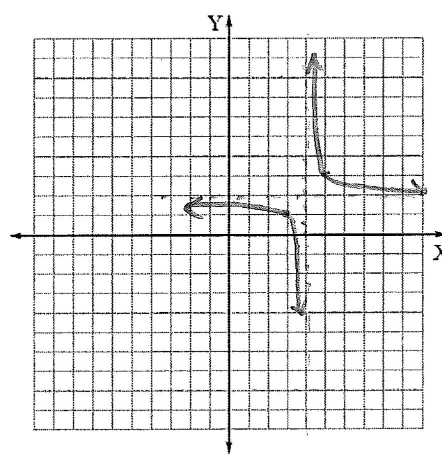


vertex @ $(-1,-5)$

y-int: $(0,-4)$

x-int: $(-1+\sqrt{5}, 0)$
$(-1-\sqrt{5}, 0)$

f. $f(x) = \frac{1}{x-4} + 2$



VA @ $x=4$

HA @ $y=2$

y-int: $(0, \frac{7}{4})$

x-int: $(\frac{7}{2}, 0)$

Exponentials and Logarithms

1. Solve for the given variable using a same bases technique.

a. $4^x = 32$

$2^{2x} = 2^5$

$2x = 5$ $\boxed{x = \frac{5}{2}}$

b. $9^{-x} = \sqrt{3}$

$3^{-2x} = 3^{1/2}$

$-2x = \frac{1}{2}$ $\boxed{x = \frac{-1}{4}}$

c. $125^{x+1} = \frac{1}{5}$

$5^{3(x+1)} = 5^{-1}$

$3x + 3 = -1$ $\boxed{x = \frac{-4}{3}}$

2. Solve for the given variable using logarithms, leave your answer in exact form.

a. $3^x = 10$

$\boxed{x = \log_3 10}$

b. $5^{x+3} = 9$

$x + 3 = \log_5 9$

$\boxed{x = -3 + \log_5 9}$

c. $2^{1-x} = 3^x$

$1 - x = \log_2 3^x$

$1 = x + x \log_2 3$

$1 = x(1 + \log_2 3)$

$\boxed{x = \frac{1}{1 + \log_2 3}}$

3. Solve for the given variable.

a. $y = \log_2 256$

$y = \log_2 2^8$

$\boxed{y = 8}$

b. $3 = \ln(13x - 1)$

$e^3 = 13x - 1$

$\boxed{x = \frac{e^3 + 1}{13}}$

c. $2 = \log_b 324$

$b^2 = 324$

$\boxed{b = +18}$

(since $b > 0$)

4. Rewrite as a single logarithm of a single argument and simplify

a. $\frac{1}{2}\log_2 x + 2\log_2 y + 3\log_2 z$

$\log_2 \sqrt{x} + \log_2 y^2 + \log_2 z^3$

$\log_2 (y^2 z^3 \sqrt{x})$

b. TYPO: 2

$2\log_3 m + 5\log_2 n - 3\log_2 mn$

$\log_2 m^2 + \log_2 n^5 - \log_2 m^3 n^3$

$\log_2 \left(\frac{m^2 \cdot n^5}{m^3 n^3}\right)$

$\log_2 \left(\frac{n^2}{m}\right)$

5. Solve for x:

a. $\log_6(x) + \log_6(x + 5) = 1$

$\log_6(x^2 + 5x) = 1$

$x^2 + 5x = 6$

$x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$\boxed{x = 1}$ * $x = -6$ is extraneous

b. $\log_3(10x + 2) - \log_3(x + 1) = 2$

$\log_3\left(\frac{10x + 2}{x + 1}\right) = 2$

$\frac{10x + 2}{x + 1} = 9$

$10x + 2 = 9x + 9$ $\boxed{x = 7}$

6. Given $\log_b 2 \approx 0.5$ and $\log_b 3 \approx 0.7$, evaluate the following:

a. $\log_b 12$

$\log_b 2^2 \cdot 3$

$2\log_b 2 + \log_b 3$

$2(0.5) + 0.7$

$\boxed{1.7}$

b. $\log_b 1.5$

$\log_b \frac{3}{2}$

$\log_b 3 - \log_b 2$

$0.7 - 0.5$

$\boxed{0.2}$

Sequences and Series

Directions: answer questions #1-4 using the sequences below

a. $5, 12, 19, 26, 33, \ldots$

b. $3, 6, 12, 24, 48, \ldots$

c. $297, 99, 33, 11, \ldots$

d. $3, -6, -15, -2$

1. Determine if the sequences above are arithmetic or geometric. If arithmetic, state the common difference. If geometric, state the common ratio.

a. Arithmetic $d = 7$

b. GEOMETRIC $r = 2$

c. GEOMETRIC $r = \frac{1}{3}$

d. ARITHMETIC $d = -9$

2. Determine an explicit equation for each of the sequences above.

a. $t_n = 5 + 7(n-1)$

b. $t_n = 3(2)^{n-1}$

c. $t_n = 297\left(\frac{1}{3}\right)^{n-1}$

d. $t_n = 3 - 9(n-1)$

3. Find the 10th term in each of the sequences above.

a. $t_{10} = 5 + 7 \cdot 9 = 68$

b. $t_{10} = 3 \cdot 2^9 = 1536$

c. $t_{10} = 297\left(\frac{1}{3}\right)^9 = \frac{11}{729}$

d. $t_{10} = 3 - 9 \cdot 9 = -78$

4. Find sum of the first 10 terms in each of the sequences above.

a. $S_{10} = \frac{10}{2}\left[5 + 68\right] = 365$

b. $S_{10} = 3\left(\frac{2^{10} - 1}{2 - 1}\right) = 3069$

c. $S_{10} = 297\left(\frac{1 - \left(\frac{1}{3}\right)^{10}}{1 - \frac{1}{3}}\right) = \frac{324764}{729}$

(≈ 445.5)

d. $S_{10} = \frac{10}{2}\left[3 + -78\right] = -375$

5. A bouncy ball is dropped from the top of the Sears Tower (1,450 feet tall). Each time it strikes the ground, it bounces up to 75% of the previous height. $\{1087.5,\ 815.625,\ 611.71875, \ldots$

   a. How high will the ball bounce after it strikes the ground for the 3rd time?

   $1450(.75)^3 = 611.71875$ feet

   b. How high will the ball bounce after it strikes the ground for the nth time?

   $1450(0.75)^n$ or $1087.5(0.75)^{n-1}$

   c. How many times does it strike the ground before its bounce is less than 6 inches?

   $1450(0.75)^n = 0.5$

   $n = 27.7 \rightarrow 28$ bounces

6. Determine the explicit equation for an arithmetic sequence with $t_2 = 10$ and $t_{29} = 91$.

   $d = \frac{91-10}{29-2} = 3$, $t_1 = 7$ so $a_n = 7 + 3(n-1)$

7. Determine sums of the following infinite geometric series:

   a. $4 + 2 + 1 + \frac{1}{2} + \ldots$

   $S = \frac{4}{1-\frac{1}{2}}$

   $S = 8$

   b. $12 + 4 + \frac{4}{3} + \ldots$

   $S = \frac{12}{1-\frac{1}{3}}$

   $S = 18$

8. Evaluate the series (sums):

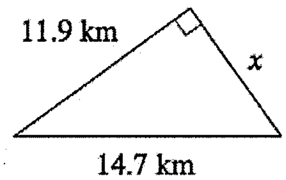
   a. $\sum_{k=1}^{5} 2(3^{k-1}) = 2\cdot3^0 + 2\cdot3^1 + 2\cdot3^2 + 2\cdot3^3 + 2\cdot3^4 = 242$

   b. $\sum_{k=1}^{10} 4k + 2 = 6 + 10 + 14 + \ldots + 42 = 240$

Geometry

1. Solve for the missing sides or angles in the triangle using Pythagorean Theorem or Special Right Triangles.

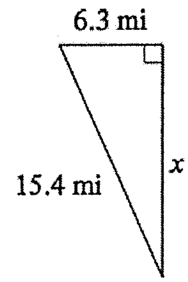
a.



$11.9^2 + x^2 = 14.7^2$

$x = \sqrt{14.7^2 - 11.9^2}$ $\quad x = 8.63$ Km

b.

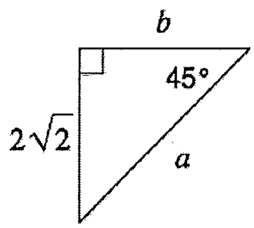


$x^2 + 6.3^2 = 15.4^2$

$x = \sqrt{15.4^2 - 6.3^2}$

$x = 14.05$ mi

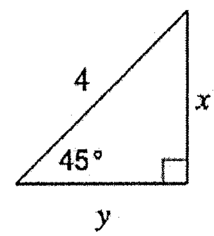
c.



$b = 2\sqrt{2}$

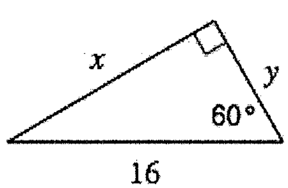
$a = 2\sqrt{2} \cdot \sqrt{2} = 4$

d.



$x = y = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

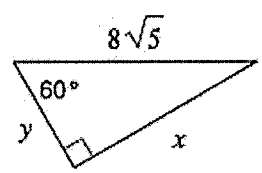
e.



$y = \frac{16}{2} = 8$

$x = 8\sqrt{3}$

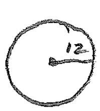
f.



$y = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$

$x = 4\sqrt{5} \cdot \sqrt{3} = 4\sqrt{15}$

2. Determine the area of the indicated shape.



a. A circle with a circumference of 24π $\quad C = 2\pi r \quad r = 12 \quad A = \pi r^2$

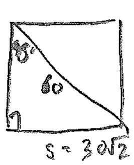
$24\pi = 2\pi \cdot r \quad A = 144\pi$

b. An equilateral triangle with perimeter 18 $\quad P = 18 \quad A = \frac{s^2\sqrt{3}}{4}$

$h = 3\sqrt{3}$ $\quad 3s = 18$

$s = 6 \quad A = 9\sqrt{3}$

c. A square with a diagonal of 60.



$A = s^2$

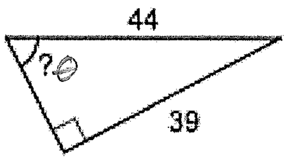
$A = (30\sqrt{2})^2$ or $A = \frac{d^2}{2}$

$A = 1800$ $\quad A = \frac{60^2}{2}$

$A = 1800$

3. Solve for the missing sides or angles in the triangle using sine, cosine, or tangent ratios (SOHCAHTOA)

a.

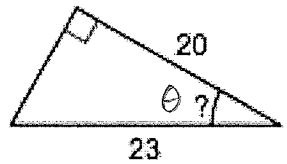


$\sin\theta = \frac{39}{44}$

$\theta = \sin^{-1}\left(\frac{39}{44}\right)$

$\theta = 62.4°$

b.

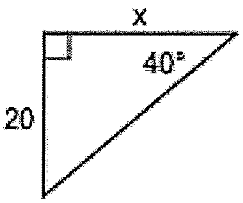


$\cos\theta = \frac{20}{23}$

$\theta = \cos^{-1}\left(\frac{20}{23}\right)$

$\theta = 29.6°$

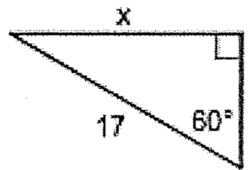
c.



$\tan 40° = \frac{20}{x}$

$x = 23.8$

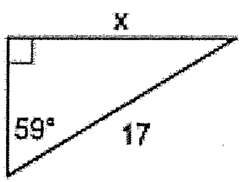
d.



$\sin 60° = \frac{x}{17}$

$x = 14.7$

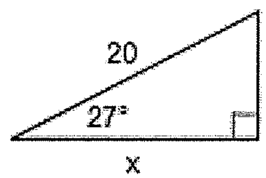
e.



$\sin 59° = \frac{x}{17}$

$x = 14.6$

f.



$\cos 27° = \frac{x}{20}$

$x = 17.8$

4. Find the volume and surface area of the indicated shape.

   a. A cylinder with a radius of 5 and a height of 12

   $V = \pi r^2 h$ $V = 300\pi$ $SA = 2\pi r^2 + 2\pi r h$ $SA = 170\pi$

   b. A sphere with a radius of 9

   $V = \frac{4}{3}\pi r^3$ $V = 972\pi$ $SA = 4\pi r^2$ $SA = 324\pi$

   c. A rectangular prism with dimensions 3x7x11.

   $V = L \times W \times h$ $SA = 2LW + 2LH + 2WH$

   $V = 231$ $SA = 263$

Quadratic and Polynomial Functions

1. Factor each polynomial completely. That is, write each as a product of prime polynomials.

a. $3x^2+7x+4$

$(3x+4)(x+1)$

b. $3x^3+x^2+12x+4$

$x^2(3x+1)+4(3x+1)$

$(x^2+4)(3x+1)$

c. t^4-22t^2+40

$(t^2-20)(t^2-2)$

d. $64x^8-16y^4$

$16(4x^8-y^4)$

$16(2x^4-y^2)(2x^4+y^2)$

e. x^3-125

$(x-5)(x^2+5x+25)$

f. $9x^2-24x+16$

$(3x-4)^2$

g. $6x^3+48$

$6(x^3+8)$

$6(x+2)(x^2-2x+4)$

h. $6x^2y-21x^2-4y+14$

$3x^2(2y-7)-2(2y-7)$

$(3x^2-2)(2y-7)$

2. Solve. Find both real and complex solutions.

a. $5x^2-x=0$

$x(5x-1)=0$

$x=\{0,\frac{1}{5}\}$

b. $x^2-4x+5=0$

$x=\frac{4\pm\sqrt{16-4(1)(5)}}{2}$ $\quad x=\{2-i,2+i\}$

c. $3(x-2)^2+48=0$

$3(x-2)^2=-48$

$(x-2)^2=-16$

$x-2=\pm 4i$

$x=\{2-4i,2+4i\}$

d. $3x^3-12x^2-36x=0$

$3x(x^2-4x-12)=0$

$3x(x-6)(x+2)=0$

$x=\{-2,0,6\}$

Radical Functions

1. Rewrite with a common denominator and simplify.

a. $\frac{x}{x-4} - \frac{3}{x+2}$

$$\frac{x(x+2)-3(x-4)}{(x-4)(x+2)}$$

$$\frac{x^2-x+12}{(x-4)(x+2)}$$

b. $\frac{5a}{2b} + \frac{6b}{5} + \frac{3}{a}$

$$\frac{25a^2+12ab^2+30b}{10ab}$$

2. Simplify the complex fraction.

a. $\dfrac{\frac{x^2-y^2}{2xy}}{\frac{1}{x}+\frac{1}{y}}$

$$\frac{\frac{(x+y)(x-y)}{2xy}}{\frac{y+x}{xy}} \rightarrow \frac{(x+y)(x-y)}{2xy} \cdot \frac{xy}{(y+x)}$$

$$\rightarrow \frac{x-y}{2}$$

b. $\dfrac{\frac{2}{x+h}-\frac{2}{x}}{h}$

$$\frac{2x-2(x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$\frac{2x-2x-2h}{xh(x+h)}$$

$$\frac{-2h}{xh(x+h)} \rightarrow \frac{-2}{x(x+h)}$$

3. Solve.

a. $\frac{1}{x-8} - 1 = \frac{7}{x-8}$

$$\frac{1}{x-8} - \frac{x-8}{x-8} = \frac{7}{x-8}$$

$$\frac{1-(x-8)}{x-8} = \frac{7}{x-8}$$

For $x \neq 8$, $1-(x-8) = 7$

$9 - x = 7$

$x = 2$

b. $\frac{x+5}{x^2+x} = \frac{1}{x^2+x} - \frac{x-6}{x+1}$

$$\frac{x+5}{x(x+1)} = \frac{1}{x(x+1)} - \frac{x(x-6)}{x(x+1)}$$

For $x \neq 0, -1$

$x+5 = 1 - x(x-6)$

$x+5 = 1 - x^2 + 6x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$ $x = \{1, 4\}$

4. Graph the rational function in the space provided. Show all intercepts, holes, and horizontal & vertical asymptotes.

a. $f(x) = \frac{2x^2+10x+12}{x^2+2x+1}$

HA: $y = \frac{2x^2}{x^2} = 2$ VA: $(x+1)^2 = 0$

$x = -1$

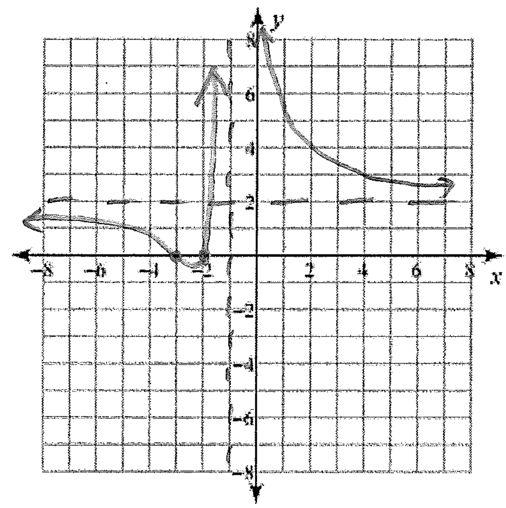
NO HOLES

$f(x) = \frac{2(x+2)(x+3)}{(x+1)^2}$

x-int: $0 = \frac{2(x+2)(x+3)}{(x+1)^2}$

$(-2,0)$ and $(-3,0)$

y-int: $f(0) = \frac{2 \cdot 0^2 + 10 \cdot 0 + 12}{0^2 + 2 \cdot 0 + 1} = 12$



b. $g(x) = \frac{x^3-16x}{-4x^2+4x+24}$

HA: none (degree in num > den)

VA: $-4(x-3)(x+2) = 0$ $x = 3, -2$

$g(x) = \frac{x(x-4)(x+4)}{-4(x-3)(x+2)}$

x-int: $0 = \frac{x(x-4)(x+4)}{-4(x-3)(x+2)}$

$(0,0)$ $(4,0)$ and $(-4,0)$

y-int: $g(0) = \frac{0}{24} = 0$